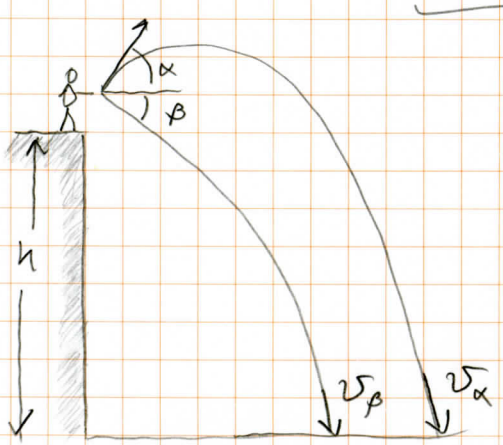
TB Pe 3.118

TWO BALLS ARE THROWN AT EQUAL SPEEDS FROM A CLIFF OF HEIGHT h. ONE IS THROWN AT AN ANGLE $+\alpha$ AND THE OTHER AT $-\beta$. SHOW THAT EACH HITS THE GROUND AT THE SAME SPEED.



$$v_{0\alpha} = v_{0\beta} = v_0$$

APPLY KINEMATICS TO FIND v's

STONE α: $v_{\alpha y}^2 = v_{0y}^2 - 2g(y - y_0)$ (with $y_0 = h$, $y = 0$)

$$= v_0^2 \sin^2\alpha + 2gh$$

$$v_{\alpha x}^2 = v_{0x}^2$$

$$= v_0^2 \cos^2\alpha$$

THE SPEED AT WHICH IT HITS IS

$$v_\alpha = \sqrt{v_{\alpha x}^2 + v_{\alpha y}^2}$$

$$= \sqrt{v_0^2 \cos^2\alpha + v_0^2 \sin^2\alpha + 2gh}$$

$$\boxed{v_\alpha = \sqrt{v_0^2 + 2gh}}$$

STONE β: $v_{\beta y}^2 = v_{0y}^2 - 2g(y - y_0)$ (with $y_0 = h$, $y = 0$)

$$= v_0^2 \sin^2\beta + 2gh$$

$$v_{\beta x}^2 = v_{0x}^2$$

$$= v_0^2 \cos^2\beta$$

THE SPEED AT WHICH THIS HIS IS

$$v_\beta = \sqrt{v_{\beta x}^2 + v_{\beta y}^2}$$

$$= \sqrt{v_0^2 \cos^2\beta + v_0^2 \sin^2\beta + 2gh}$$

$$\boxed{v_\beta = \sqrt{v_0^2 + 2gh}}$$

BY INSPECTION, $\boxed{v_\alpha = v_\beta = \sqrt{v_0^2 + 2gh}}$ QED!